Optimal Road Pricing with Congestion and Fund Procurement

Paper Identification number: SCS12-034
Hidenori IKESHITA[1], Hisayoshi MORISUGI[2], Atsushi FUKUDA[3]

[1]Transportation Engineering and Socio Technology,
Graduate School of Science and Technology, Nihon University
Telephone +81-47-469-5355, Fax +81-47-469-5355
E-mail: cshi11002@g.nihon-u.ac.jp

[2]Department of Transportation Engineering and Socio Technology,
College of Science and Technology, Nihon University
Telephone +81-47-469-5355, Fax +81-47-469-5355
E-mail: morisugi.hisayoshi@nihon-u.ac.jp

[3]Department of Transportation Engineering and Socio Technology,
College of Science and Technology, Nihon University
Telephone +81-47-469-5355, Fax +81-47-469-5355
E-mail: fukuda.atsushi@nihon-u.ac.jp

Abstract

This paper discussed the optimal road pricing, which maximizes social surplus under a user equilibrium condition with imperfect substitution assumption for route choice in a transportation network with many nodes and links. So far several attempts to integrate road pricing theory as economic measure and transportation network equilibrium analysis have been made but they are inconsistent with the economic theory. First, we took account of the marginal cost of funding from road pricing. In general, congestion pricing is based on the principle of marginal cost pricing which equal to the difference between marginal social and marginal private cost. In this study, we defined the social welfare function which consists of a sum of indirect utility function as consumers' surplus and road pricing revenue as producer's surplus. Following this definition, we formulated the user behaviour maximizing a quasi-linear utility with imperfect substitution between any route as defined the consumer's utility function of the road user's under the constraint of budget and time. Then we get the demand function and the indirect utility function.

Regarding consumer's utility function, we could derive the equilibrium condition in the same manner as stochastic user equilibrium with fixed transport demand and revealed the objective function. And then, we derived optimal road pricing for first best and distorted second best world. Finally, our formulation could derive an optimal road pricing level with considering the public welfare loss of fiscal resources procurement and road congestion.

Keywords: Road pricing, Congestion, Procurement of funds, social surplus, user equilibrium

1. Introduction

Road pricing is widely recognized among policy-maker and planners in many countries to relieve road congestion, which offers important economic benefits. Road pricing is direct charges levied for the use of busy roads and is to cover the public cost of building and maintaining roadway infrastructure. These road pricing schemes have been proposed, implemented to relieve road congestion in many cities, for example, Singapore, Hong Kong and London and also been investigating the policy experience with road pricing schemes. [1] But in reality, the problem is how to determine the toll charge and is where to obtain financing of road infrastructure.

To this end, from a practical perspective, efficient pricing models are considered for introduction on congested urban road networks. In addition, much research has focused on empirical aspects, policy experiences and environmental issues of road pricing. However, transportation network equilibrium analysis, which has been done as a practice, is inconsistent with the economic theory of road pricing. And few researchers have addressed the problem of the toll charge level with financing of road infrastructure in a simple network.

The purpose of this study is to discuss and formulate the optimal road pricing which maximizes social surplus under a user equilibrium condition in a transportation network with many nodes and links and show that in our formulation could derive an optimal road pricing level with considering the public welfare loss of fiscal resources procurement and road congestion.

2. Methodology

As a practical aspect of road pricing policy analysis, the model requires to clearly state a charge system of road pricing and to clarify the transportation network model of the road supply-side. From the theoretical aspect, road pricing is an application of theories concerning internalization of an external diseconomy to road congestion and it is one of typical amalgamations of economic theories and practical problems. [2] In the majority of the road pricing studies employ a model of road transportation equilibrium problem for example, Mun [3] and Yang et al. [4] Those models have generality and practicality, but it is necessary to develop the model under theoretical individual behaviour, or equivalently, behavioural theory of micro-economics. To address this problem, we formulated the user behaviour to derive the optimal road pricing in a transportation network with many nodes and links.

2.1 Outline of Model

This study assumed that the condition of optimal road pricing level as shown in below.

1. The planner may impose the toll fee of each link to road users.
2. Road users implement traffic volume assignment of path flow to maximize their utility.
3. Road users recognize the impact of their behaviour has on traffic congestion.
4. The link duration is described as a monotonically increasing convex function of link traffic volume.
5. Total demand (trip distribution) is not limited.

As mentioned above conditions, we formulate optimal road pricing level which is applied under conditions of network equilibrium. First, we formulate the formula for efficient optimal road pricing level which maximizes social surplus. That is, we defined the social surplus which consists of a sum of indirect utility function as consumers' surplus and road pricing revenue as producer's surplus. Then, we defined the consumer's utility function of the road user's under the constraint of budget and time. And then we will find that our formulation could derive an optimal road pricing level with considering road congestion and the public welfare loss of fiscal resources procurement.

2.2 Model

2.2.1 Social Welfare Function

A social welfare function is defined as shown in equation(1). This is the sum of indirect utility function as consumer's surplus and welfare loss of taxpayer which is a construction cost minus toll charge revenue as producer's surplus.

$$W = V + MCF\left[I - P_a x_a\right] \qquad (1)$$

where MCF is marginal cost of fund procurement and is a negative value. The issues of pricing, there is considered between construction cost of a link and its toll charge. The construction cost of the link is *I*, and its fund comes from toll charge revenue of link *a*. For the time being, fuel tax is not treated in this study. And it is assumed that toll charge revenue of each link use for its construction cost.

As mentioned above condition, optimal road pricing level which maximizes social welfare function *W* is satisfied the equation(2).

$$\frac{\partial W}{\partial P_a} = \frac{\partial V}{\partial P_a} - MCF\left(x_a + P_a\frac{\partial x_a}{\partial P_a}\right) = 0 \qquad (2)$$

where $\left(x_a + P_a\frac{\partial x_a}{\partial P_a}\right)$ means that the marginal revenue of link *a*. To identify function *V*, we will derive the consumer's utility function *V* in the next section.

2.2.2 Consumer's Utility Function

First, we defined the consumer's utility function *V* of the road user's is given by

$$\max_{z, f_k^{rs}, l} V = z + u\left(f_{k=1}^{rs=1}, f_{k=2}^{rs=1}, \cdots, f_{k=n(m)}^{rs=m}, l \right) \qquad (3)$$

and the constraint of budget and time are subject to

$$z + \sum_{rs=1}^{m}\sum_{k=1}^{n} P_k^{rs} f_k^{rs} = wL + y \qquad (4)$$

$$l + \sum_{rs=1}^{m}\sum_{k=1}^{n} t_k^{rs}(\bar{f}) f_k^{rs} + L = T \qquad (5)$$

where P_k^{rs} is the price P of the route k between OD pair rs, w is the wage rate, L is the labour hours, y is asset income, t_k^{rs} is the duration of the path k between the OD pair rs (Function of path flow traffic volume vector f) and T is the total time available. And $\bar{f}$ is path flow traffic volume vector for equilibrium, which given condition for road users. That is, it assumed that they disregard the impact their traffic has on others traffic condition. This treatment is described the externality of road congestion.

Let f_k^{rs}, P_k^{rs} and $t_k^{rs}(\bar{f})$ denotes the flow on path k between OD pair rs and x_a the flow on link a. The relationship between link flows and path flows can be expressed by

$$x_{a'} = \sum_{rs}\sum_{k} \delta_{a',k}^{rs} f_k^{rs} \qquad (6)$$

$$P_k^{rs} = \sum_{a'} \delta_{a',k}^{rs} P_{a'} \qquad (7)$$

$$t_k^{rs}(\bar{f}) = \sum_{a'} \delta_{a',k}^{rs} t_{a'}(\bar{x}_{a'}) \qquad (8)$$

where $\delta_{a,k}^{rs}$ is equal to 1 if link a is on path k and 0 otherwise, a' is all link which including link a and x_a of $t_a(\bar{x}_a)$ is total traffic volume which is given from viewpoint of individual.

To obtain the demand function and indirect utility function from above conditions, we apply a Lagrange's method of undetermined multipliers as

$$\begin{aligned} L &= z + u\left(f_{k=1}^{rs=1}, f_{k=2}^{rs=1}, \cdots, f_{k=n(m)}^{rs=m}, l \right) \\ &+ \lambda\left(wT + y - z - wl - \sum_{a}\left(P_a + wt_a(\bar{x}_a)\right)x_a \right) \\ &+ \sum_{a'} \mu_{a'}\left(x_{a'} - \sum_{rs=1}^{m}\sum_{k=1}^{n} \delta_{a',k}^{rs} f_k^{rs} \right) \end{aligned} \qquad (9)$$

By evaluating the first order derivative of the Lagrangian with respect to the decision variables, we have the following first order optimality conditions:

$$\frac{\partial L}{\partial z} = 1 - \lambda = 0 \qquad (10)$$

$$\frac{\partial L}{\partial l} = \frac{\partial u\left(f_{k=1}^{rs=1}, f_{k=2}^{rs=1}, \cdots, f_{k=n(m)}^{rs=m}, l \right)}{\partial l} - \lambda w = 0 \qquad (11)$$

$$\begin{aligned} \frac{\partial L}{\partial f_k^{rs}} &= \frac{\partial u\left(f_{k=1}^{rs=1}, f_{k=2}^{rs=1}, \cdots, f_{k=n(m)}^{rs=m}, l \right)}{\partial f_k^{rs}} \\ &- \sum_{rs=1}^{m}\sum_{k=1}^{n} \mu_a \delta_{a,k}^{rs} = 0 \end{aligned} \qquad (12)$$

$$\frac{\partial L}{\partial x_a} = -\lambda\left(P_a + t_a(\bar{x}_a)\right) + \mu_a = 0 \qquad (13)$$

$$\frac{\partial L}{\partial \lambda} = wT + y - z - wl - \sum_{a}\left(P_a + t_a(\bar{x}_a)\right)x_a = 0 \qquad (14)$$

$$\frac{\partial L}{\partial \mu_a} = x_a - \sum_{rs=1}^{m}\sum_{k=1}^{n} \delta_{a',k}^{rs} f_k^{rs} = 0 \qquad (15)$$

From above first order conditions, path flow demand function f_k^{rs} and leisure demand function l, we obtain

$$f_k^{rs} = f_k^{rs}\left(1, w, P_k^{rs} + wt_k^{rs}(\bar{f})\right) \qquad (16)$$

$$l = l\left(1, w, P_k^{rs} + wt_k^{rs}(\bar{f})\right). \qquad (17)$$

Then, the indirect utility function V, we get

$$V = wT + y + v\left(1, w, P_k^{rs} + wt_k^{rs}(\bar{f})\right). \qquad (18)$$

In general, it is noted that to be true the Roy's identity of indirect utility function V, we obtain

$$-\frac{\partial V}{\partial\left(P_k^{rs} + wt_k^{rs}(\bar{f})\right)} = f_k^{rs}\left(l, w, P_k^{rs} + wt_k^{rs}(\bar{f})\right). \qquad (19)$$

Following above consumer's utility function, we derive the equilibrium condition in the same manner as stochastic user equilibrium with fixed transport demand.

3. Optimal Road Pricing on a Link

We derive the optimal road pricing level of link *a* which to maximize the previously mentioned social welfare function *W*. First, we derived the first-best pricing in the case where road user no choice of a tolled route and the second-best pricing in the case where road users can choose between a tolled and untolled route.

3.1 First-best Pricing

First-best pricing formulation, we get

$$dw = \sum_a \frac{\partial W}{\partial P_a} dP_a$$

$$= \sum_a \left(\frac{\partial V}{\partial P_a} - MCF \left(x_a + \sum_{a'} \left(P_{a'} \frac{\partial x_{a'}}{\partial P_a} \right) \right) \right) dP_a$$

$$= \begin{pmatrix} -x_a \left(1 + w \frac{\partial t_a(x_a)}{\partial x_a} \frac{\partial x_a}{\partial P_a} \right) \\ -\sum_{a' \neq a} x_{a'} \left(w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} \frac{\partial x_{a'}}{\partial P_a} \right) \end{pmatrix} dP_a \qquad (20)$$

$$- MCF \left(x_a + \sum_{a' \neq a} \left(P_{a'} \frac{\partial x_{a'}}{\partial P_a} \right) \right) dP_a$$

$$- MCF \left(\frac{\partial x_a}{\partial P_a} \right) P_a dP_a$$

$$\therefore P_a = \left(\frac{ \begin{pmatrix} -x_a \left(1 + w \frac{\partial t_a(x_a)}{\partial x_a} \frac{\partial x_a}{\partial P_a} \right) \\ -\sum_{a' \neq a} x_{a'} \left(w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} \frac{\partial x_{a'}}{\partial P_a} \right) \end{pmatrix} }{ MCF \frac{\partial x_a}{\partial P_a} } \right) \qquad (21)$$

$$- MCF \left(x_a + \sum_{a' \neq a} \left(P_{a'} \frac{\partial x_{a'}}{\partial P_a} \right) \right)$$

If the marginal cost of public fund equal to minus one, optimal pricing level $P_{a'}$, we obtain

$$P_{a'} = w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} x_{a'} \qquad (22)$$

This study assumes that the network consisting of many links and many nodes. Therefore, optimal road pricing has considered necessary that all link information. However, the equation (22) is implied that optimal road pricing level on each link could levied by observed traffic volume and how durations change depending on traffic volume. It coincides with the simplest optimal pricing solution of a simple link.

3.2 Second-best Pricing

In this study, second-best is to optimize pricing levels and other links were a set price level. Second-best pricing formulation, we get

$$\frac{\partial W}{\partial P_a} = \frac{\partial V}{\partial P_a} - MCF \left(x_a + P_a \frac{\partial x_a}{\partial P_a} \right)$$

$$= -x_a \left(1 + w \frac{\partial t_a(x_a)}{\partial x_a} \frac{\partial x_a}{\partial P_a} \right)$$

$$- \sum_{a' \neq a} x_{a'} \left(w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} \frac{\partial x_{a'}}{\partial P_a} \right) \qquad (23)$$

$$- MCF \left(x_a + P_a \frac{\partial x_a}{\partial P_a} \right)$$

$$\therefore P_a = - \frac{ \left[\begin{array}{l} x_a \left(1 \Big/ \frac{\partial x_a}{\partial P_a} + w \frac{\partial t_a(x_a)}{\partial x_a} \right) \\ + \sum_{a' \neq a} x_{a'} \left(w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} \frac{\partial x_{a'}}{\partial P_a} \right) \Big/ \frac{\partial x_a}{\partial P_a} \end{array} \right] }{MCF} - x_a \Big/ \frac{\partial x_a}{\partial P_a} \qquad (24)$$

If the marginal cost of public fund equal to minus one, optimal pricing level P_a, we obtain

$$P_a = w \frac{\partial t_a(x_a)}{\partial x_a} x_a$$

$$- \sum_{a' \neq a} x_{a'} \left(P_{a'} - w \frac{\partial t_{a'}(x_{a'})}{\partial x_{a'}} x_{a'} \right) \frac{ \frac{\partial x_{a'}}{\partial P_a} }{ \frac{\partial x_a}{\partial P_a} } \qquad (25)$$

On equation (25) second term is all the other link pricing level $P_{a'}$ adjust the link *a* pricing level to minimize distorted which has lost touch with social marginal cost. In this case needs information on all links.

4. Conclusion

In this study, we formulated the optimal road pricing level on network equilibrium conditions.

First, we defined the consumer's utility function as road users, and then we derived demand function and indirect utility function.We confirmed the formulated user behaviour on user equilibrium condition between existing equilibrium models. That is, in this study, it assumed that road user accepted the road congestion as externality and disregards the impacts on other effects.

Next, we derived the first-best and second-best efficient pricing formulation to maximize social welfare. As a result, first-best pricing is only related to traffic volume of target link. And second-best is considering to minimize distorted which has lost touch with social marginal cost.

References

[1] A. Anas and R. Lindsey (2011) Reducing Urban Road Transportation Externalities: Road Pricing in Theory and Practice, *Review of Environmental Economics and Policy*, Vol.5, Issue 1, pp.66-88

[2] Kenzo TAKEUCHI (1997) ROAD PRICING: Problems and Its Future From an Economist's Viewpoint, *IATSS Research*, Vol.21, No.2, pp. 91-99.

[3] Se-il Mun (2005) Theory of Traffic Congestion and Policy（交通混雑の理論と政策）, Toyo Keizai (in Japanese).

[4] Hai Yang and Hai-Jun Huang (2005) Mathematical and Economic Theory of Road Pricing, Elsevier Science.